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RESEARCH STUDY  
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# MODEL FOR WIND FLOW IN AN IDEALIZED VEGETATIVE CANOPY

(METEOROLOGICAL RESEARCH NOTES NO. 5)

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ERDAA-MET-7-63  
Research Study  
June 1963

A MODEL FOR WIND FLOW IN AN IDEALIZED VEGETATIVE CANOPY

By

R. M. Cionco, W. D. Ohmstedt, and J. F. Appleby

OBJECTIVE

The objective of Task 1-A-0-11001-B-021-08, "Micrometeorology (ERDAA)," is to conduct studies dealing with the physical processes involved in the exchange of energy between the atmosphere and the earth's surface. Through such basic research, increased knowledge of atmospheric processes will result which can be used by applied research activities to improve the design and test criteria for Army weapons and systems and to improve weather observing and forecasting for Army tactical operations.

AUTHORITY

Authority for this task is contained in letter, OCSigO, SIGRD-8b-5, dated 13 August 1957, "Proposed Coordinated Signal Corps Meteorological Program."

## SUMMARY

The purpose of the report is to present the results of a study which examines the characteristic parameters of wind flow in an idealized vegetative canopy. Equations characterizing the wind flow in and above the vegetative canopy are derived and compared with data collected in and above various crops.

Observations taken in a corn field indicate a marked similarity to the flow derived for the ideal canopy. Although the utility of the approach was not proven, the concept does seem to show promise.

It is hoped this report will stimulate thought and investigation which will ultimately lead to the objective of expressing the aerodynamic roughness effects in terms of vegetative characteristics.

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**RESEARCH STUDY**  
**A MODEL FOR WIND FLOW IN AN IDEALIZED VEGETATIVE CANOPY**

**DA Task 1-A-0-11001-B-021-08**

**INTRODUCTION**

To a large extent, micrometeorology is a science devoted to the evaluation of physical processes at the boundary or interface between the earth and the atmosphere. With regard to atmospheric turbulence within the boundary layer, the most important single characteristic of the boundary is its effective aerodynamic roughness. Over land surfaces, with few exceptions, the boundary is considered to be fully rough; that is, a laminar sublayer is essentially nonexistent. Under natural conditions, the roughness over land is primarily associated with vegetation, although, in urban areas man-made structures may dominate.

For the general application of micrometeorological prediction models, it would be most advantageous if the aerodynamic roughness effect of the boundary could be expressed in terms of the height, density and drag characteristics of the roughness elements. To date, no such goal has been achieved. Consequently, it was the objective of the study reported here to investigate the turbulent transfer of momentum within a vegetative canopy and to attempt to develop a preliminary model which expressed the aerodynamic roughness effects in the terms of characteristics of the vegetation.

**BACKGROUND**

For practical reasons, it has been the goal of the majority of workers in turbulence research to express the turbulent transfers in terms of mean quantities of the velocity and its derivatives rather than the variances and covariances of the turbulent fluctuations which are known as the Reynolds stresses. Although the latter are fundamentally sound, they consist, in essence, of only statistical quantities which describe the fluctuation kinematics. It is the intent here to continue the practice of expressing the dynamic characteristics of the turbulent transfer in terms of the mean quantities.

Many of the developments of models for turbulent transfer have come about through reasoning on dimensional grounds and lean heavily on empirical quantities and relationships. There has been only limited success in pursuing the problem in this manner and it is safe to say that a general model for atmospheric turbulence remains remote. On the positive side, however, remarkable success has been achieved in special cases with very simple models. Fore-

most in this area is the mixing-length hypothesis attributed to Prandtl. This concept will be used later in the discussion of the turbulent transfer within the vegetative canopy, so we shall review briefly the characteristics of the mixing-length hypothesis for fully rough, steady flow at very high Reynolds numbers. The basic relationship of this hypothesis is as follows:

$$\frac{\tau}{\rho} = u_*^2 = \ell^2 \left( \frac{\partial u}{\partial z} \right)^2 \quad (1)$$

where  $\tau$  is the shearing stress,  $\rho$  is the density,  $u_*$  is the friction velocity,  $\ell$  is the mixing length,  $u$  is the mean velocity parallel to the boundary, and  $z$  is the space coordinate normal to the boundary. If the analogy with laminar flow is invoked such that the shearing stress is proportional to the shear, we obtain:

$$\frac{\tau}{\rho} = u_* \ell \frac{\partial u}{\partial z} = K \frac{\partial u}{\partial z} \quad (2)$$

where  $K$  is the eddy viscosity.

The behavior of the mixing length in the vicinity of a rough boundary can best be demonstrated by citing observations of the boundary layer of a flat plate at very high Reynolds numbers. These observations show that, near the boundary, the friction velocity is essentially constant and the shear is inversely proportional to the height above the boundary. Consequently, the mixing length must be proportional to the height above the boundary. This is expressed as follows:

$$\ell = kz \quad (3)$$

where  $k$  is von Karman's constant. The real success of the mixing-length hypothesis lies in the fact that  $k$  is, indeed, a constant which is independent of the scale of flow; that is, equation (3) is equally applicable to the atmospheric surface boundary layer as well as liquid flow in pipes.

Combining equation (1) with equation (3) and integrating results in the following velocity profile relationship:

$$\frac{u}{u_*} = \frac{1}{k} \left[ \ln z + c \right] \quad (4)$$

where  $c$  is a constant of integration. It is through this constant of integration that the roughness enters the equations of turbulent transfer. The most comprehensive quantitative expression for  $c$  is due to Nikuradse, (Reference 1). He conducted extensive experiments on flow through pipes roughened by tightly packed sand grains of

characteristic size  $k_s$ . For fully rough flow, his results showed that  $c = \ln(30/k_s)$ .

The expression "fully rough flow" requires explanation. For pipe flow, it is useful to define a drag coefficient ( $C_D$ ) as follows:

$$u_{*0}^2 = \tau_0 / \rho = \frac{1}{2} C_D \bar{U}^2 \quad (5)$$

where  $\tau_0$  is the shearing stress at the boundary and  $\bar{U}$  is the mean flow velocity through the pipe. A fully rough flow is one in which  $C_D$  is independent of the Reynolds number,  $\bar{U}d/\nu$ , where  $d$  is the diameter of the pipe and  $\nu$  is the kinematic viscosity. The fully rough regime occurs for Reynolds numbers exceeding a certain critical value. This critical Reynolds number increases for decreasing relative roughness ( $k_s/d$ ).

The experiments of Nikuradse incorporated tightly packed sand grains and no attempt was made to vary the density of the roughness elements. Schlichting (Reference 2) conducted experiments analogous to those of Nikuradse but with varying densities of regularly shaped roughness elements (spheres and hemispheres). He expressed his results in terms of the effective sand grain roughness of Nikuradse. As would be anticipated, the effective sand grain roughness increased with increasing density of roughness elements. One interesting exception to this rule was that, for the most dense arrangement of spheres, the effective sand grain roughness was less than for the same spheres with less packing. This result can be interpreted as meaning the spheres were so tightly packed that the fluid "saw" a new zero level displaced inward from the actual wall of the pipe; that is, the fluid recognized only a portion of the actual height of the spheres in terms of roughness.

In the past, the treatment of roughness effects on the atmospheric boundary layer have largely followed the precedents set in the above consideration of fully rough pipe flow but with some change of parameters. The most general expression\* for the wind profile equation is as follows:

$$\frac{u}{u_*} = \frac{1}{k} \left[ \ln \frac{z - D}{z_0} \right] \quad (6)$$

\* Equation (6) is applicable only to regions immediately above the boundary where the shearing stress is constant with height and effects of buoyancy in diabatic conditions are negligible. This condition is generally satisfied in the first two or three meters above the top of the roughness elements.

where  $D$  is the zero-plane displacement and  $z_0$  is the roughness length.

The behavior of the zero-plane displacement and the roughness length is well demonstrated by the results of a unique set of experiments reported by Kutsbach (Reference 3). These experiments were somewhat analogous to those of Schlichting described above. The friction velocity, zero-plane displacement, and roughness length were evaluated for an ensemble of bushel baskets of varying density patterns placed on an ice-covered lake. The results showed that the roughness length increased approximately linearly with increasing density of baskets, while the zero-plane displacement increased with the basket density to the 0.3 power. Although there was considerable scatter in the zero-plane displacement relationship, the results indicate a rather orderly variation of the roughness parameters with the density of baskets. In another experiment, the density of the bushel baskets was maintained constant, but the height of the obstacles was varied by using successively one basket and then doubling the height by placing one basket on top of another. As a result, the zero-plane displacement increased by a factor of 2.5. The difference from a factor of 2 is likely due to a change of shape of the roughness element from one inverted basket to that of an inverted basket placed on top of an up-right basket. In a qualitative sense this result is compatible with the results of Schlichting which indicated that spheres are rougher than hemispheres.

The bushel basket results indicate an orderly variation of the roughness parameters with the density and height of rigid roughness elements. However, the results are considerably less definitive when the ensemble of roughness elements is a uniform field of vegetation. This is clearly indicated in results published by Tan and Ling (Reference 4) and Stoller and Lemon (Reference 5) for wind profiles over fields of wheat, alfalfa, and corn. Briefly, their results with corn show that, for a given crop density and height, the zero-plane displacement decreases and the roughness length increases with increasing wind speed; in alfalfa the opposite is true with the unexpected result that the shearing stress is essentially independent of wind speed. It is not difficult to imagine the cause of these changes. Alfalfa is limber so with increasing wind speed the stems bend over and the leaves tend to orient themselves along lines of least resistance. The laying over of the plants partially seal off the canopy from penetration of the mainstream above. The result is decreasing roughness and increasing zero-plane displacement with increasing wind speed for a limber vegetative canopy. Corn is characterized by semi-rigid stalks and limber but resilient leaves. Increasing wind speed results in greater penetration into the somewhat open canopy and a flapping of the leaves. The result is decreasing zero-plane displacement and increasing roughness with increase in wind speed in corn. So long as the roughness length and the zero-plane displace-

ment are the sole parameters used, it would appear that little more than the qualitative reasoning applied above could be invoked to characterize the roughness effects of a vegetative canopy. The point is that the roughness length and the zero-plane displacement are merely convenient parameters to specify the boundary condition of the logarithmic wind profile and are only vaguely related to the actual physical processes involved in the roughness effects. For this reason an entirely new approach was utilized in this study to evaluate the roughness effects of a vegetative canopy. The intent was to develop a model for turbulent transfer within the canopy which expressed the boundary conditions for the logarithmic wind profile directly in terms of the height, density and drag characteristics of the vegetation.

### THE IDEAL CANOPY

For preliminary considerations of turbulence within the vegetation, let us consider the steady-state adiabatic turbulent energy equation (see Townsend Reference 6) in the following form:

$$\sum_{j=1}^3 \left\{ \sum_{i=1}^3 \left[ \overline{u_i' u_j'} \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\bar{u}_j}{2} \frac{\partial \overline{u_i'^2}}{\partial x_j} \right] \right\} + \epsilon = 0$$

$i, j = 1, 2, 3$  (7)

where the subscripts designate the three components in a rectangular coordinate system,  $u_{ij}'$  represents the turbulent velocity component in the  $i, j$  direction,  $\bar{u}_{ij}$  is the local mean velocity component in the  $i, j$  direction,  $\epsilon$  is the local dissipation and a bar designates a time average. Equation (7) is derived from the Navier-Stokes equation and is assumed to apply to any small localized region of the fluid within the vegetative canopy.

To apply equation (7) in a practical way, let us define a volume  $V$  which is a horizontal slice containing a representative quantity and arrangement of leaves and stalks. For this volume, we can define a gross mean velocity  $U$  by integration over the volume of the localized mean velocity components  $\bar{u}_i$ . It is reasonable to assume that  $U$  is horizontal and parallel with the ambient mean velocity above the vegetative canopy. Furthermore, we define a length scale  $L$  which for now will be assumed to characterize the average leaf-stalk arrangement within the volume  $V$ . Next, we non-dimensionalize the terms of equation (7) as follows:  $v_{ij}' = u_{ij}'/U$ ,  $\bar{v}_{ij} = \bar{u}_{ij}/U$  and  $y_{ij} = x_{ij}/L$  with the result that equation (7) becomes:

$$\frac{U^3}{L} \sum_{j=1}^3 \left\{ \sum_{i=1}^3 \left[ \overline{v_i' v_j'} \frac{\partial \bar{v}_i}{\partial y_j} + \frac{\bar{v}_j}{2} \frac{\partial \overline{v_i'^2}}{\partial y_j} \right] \right\} + \epsilon = 0$$

(8)

Integrating equation (8) over the volume V, we obtain the following expression for the mean dissipation:

$$\bar{\epsilon} = \left( \frac{C'_e}{L} \right) U^3 \quad (9)$$

where the dissipation coefficient  $C'_e$  of the vegetation within the volume V is defined as:

$$C'_e = - \frac{1}{V} \int_V \int_V \sum_{j=1}^3 \left\{ \sum_{i=1}^3 \left[ \overline{v'_i v'_j} \frac{\partial \bar{v}_i}{\partial y_j} + \frac{\bar{v}_j}{2} \frac{\partial \overline{v'^2_i}}{\partial y_j} \right] \right\} dx dy dz \quad (10)$$

To complete the picture, we obtain the average values of the Reynolds stresses within the volume V by like integration. If we assume, as usual, that the horizontal derivatives of the gross mean quantities of the volume V are negligible and remembering that  $u_*^2 = -\overline{u'w'}$ , then equation (7) can be written to describe the mean dissipation of the volume V as follows:

$$u_*^2 \frac{\partial U}{\partial z} = \bar{\epsilon} = (C'_e/L) U^3 \quad (11)$$

Rearranging terms, we obtain the following equation:

$$\left( \frac{u_*}{U} \right)^2 \frac{\lambda(\ln U)}{\partial z} = \frac{C'_e}{L} \quad (12)$$

Equation (12) represents a preliminary expression of turbulent transfer of momentum within a vegetative canopy. In its derivation, we have introduced two terms—the dissipation coefficient  $C'_e$  and the scale length L. The usefulness of equation (12) depends upon whether these two factors can be evaluated experimentally and expressed empirically as unique functions of the density and structure (or species) of vegetation.

A dissipation coefficient can be defined for flow through a fully rough pipe in the same manner as was done for the drag coefficient. In the case of pipes, the scale length L is the diameter of the pipe. For sufficiently high Reynolds numbers, the dissipation coefficient is independent of Reynolds number, but its magnitude depends upon the relative roughness of the pipe ( $k_s/d$ ). It should be noted that equation (9), which defines the dissipation coefficient, differs from equation (5) which defined the drag coefficient by inclusion of the scale length L. Thus, at very high Reynolds numbers, greater dissipation occurs in small pipes than in larger pipes, even though the mean velocities and shearing stresses may be the same. If we apply this reasoning to a vegetative canopy, we would assume that the scale length in some way measures the average distance between

adjacent parts of the plants. If this is the case, we would expect a higher degree of local dissipation associated with closely packed plants such as grass or alfalfa as opposed to more loosely arranged canopies such as a corn field or forest.

The term  $(u_*/U)$  is somewhat analogous to the intensity of the turbulence. It is not unreasonable to assume that the turbulence intensity is closely correlated with the drag coefficient (see equation (5)) and thus the dissipation coefficient.

For an ideal canopy, let us assume that the dissipation coefficient and the scale length  $L$  are constant with height in the canopy. This is equivalent to assuming that the density, size, and arrangement of the leaves and stalks are "essentially" invariant within the canopy. Because of the inferred proportionality of the turbulence intensity to dissipation coefficient, we further characterize the ideal canopy as having a constant turbulence intensity as expressed by  $(u_*/U)$ . Consequently, the term  $\partial(\ln U)/\partial z$  in equation (12) must also be a constant. Thus, the velocity profile of the ideal canopy must be an exponential function of height.

The effect of roughness on the surface boundary layer becomes evident only in the integrated equation which defines the velocity profile. The profile equation is derived by means of the mixing-length hypothesis. Equation (12) was derived independent of the mixing-length hypothesis, and therefore is somewhat incompatible with the more common approach to turbulent transfer.

Ordway, Ritter, Spence, and Tan (Reference 7) have suggested that the turbulent transfer within the vegetative canopy can be pursued along the same lines as the surface boundary layer provided allowance is made for the loss of momentum to the leaves and stalks. On heuristic grounds, they proposed that this momentum loss was proportional to the square of the local mean velocity. This is analogous to equation (5) which defined the drag coefficient for fully rough flow through a pipe. Further, for steady state conditions with no advective terms, the local loss of momentum must equal the convergence of momentum transport. Thus, for turbulent transfer within the vegetative canopy, the following equation results:

$$\frac{\partial}{\partial z} \left[ \frac{\tau}{\rho} \right] = \frac{\partial}{\partial z} \left[ K \frac{\partial U}{\partial z} \right] = SU^2 \quad (13)$$

where the proportionality constant  $S$  can be further subdivided as follows:

$$S = \frac{1}{2} C_D' A \quad (14)$$



where  $A$  is the effective aerodynamic surface area of the vegetation per unit volume, and  $C'_D$  is the drag coefficient of the leaf-stalk configuration. Ordway et al assumed that  $C'_D$  is independent of Reynolds number. Further, they assumed that the eddy viscosity was constant within the vegetative canopy. In a later report, Tan and Ling allowed for a variation of the eddy viscosity and suggested that it might increase linearly with height as it does in the surface boundary layer. It is at this point where our views are different from those of Tan and Ling. Through equation (1) and (2) we saw that  $K = u_* \ell$ . In the surface boundary layer,  $u_*$  is nearly constant while  $K$  and  $\ell$  increase linearly with height. However, down in the canopy, changes in  $K$  should be largely due to changes in  $u_*$  since due to the restrictive action of the leaves and stems the mixing length within the canopy could be nearly constant. Consequently, we set out to determine the size and variation of the mixing length within a vegetative canopy.

In consonance with the ideal canopy previously defined, let us assume the following equation to be valid for the mean velocity within the vegetative canopy:

$$\frac{\partial(\ln U)}{\partial z} = -\frac{h}{U} \frac{\partial U}{\partial z} = a \quad (15)$$

where  $a$  is a constant and  $h$  is the height of the canopy. If  $x$  is defined as a dimensionless height equal to  $z/h$  then from equation (15) we further observe that  $\partial^2(\ln U)/\partial x^2 = 0$  and  $U = U_h \exp[a(x-1)]$  where  $U_h$  is the mean wind velocity at the top of the canopy. Remembering that  $\frac{1}{U} \frac{\partial^2 U}{\partial x^2} = \frac{\partial^2(\ln U)}{\partial x^2} + \left[ \frac{\partial(\ln U)}{\partial x} \right]^2$ , we find that equation

(13) reduces to:

$$\frac{\partial K}{\partial x} + aK = \frac{h^2 S U}{a} = \frac{h^2 S U_h}{a} \exp[a(x-1)] \quad (16)$$

Equation (16) can be solved analytically with the result that

$$KU = K_h U_h - \frac{h^2 S}{2a^2} (U_h^2 - U^2) \quad (17)$$

Since  $U_*^2 = K \frac{\partial U}{\partial x} = \frac{KU_a}{h}$ , we find the following for the friction velocity:

$$U_*^2 = (U_*)^2_h - \frac{hS}{2a} (U_h^2 - U^2) \quad (18)$$

Furthermore,  $U_* = \ell \frac{\partial u}{\partial z} = \frac{\ell_c U_a}{h}$  according to equation (1) so  $\ell_c = \frac{h U_*}{a U}$

$$\text{or } \ell_c^2 = \frac{h^2}{2a^2} \left[ \frac{hS}{a} + \left( 2 \left( \frac{U_{*h}}{U_h} \right)^2 - \frac{hS}{a} \right) \left( \frac{U_h}{U} \right)^2 \right] = \frac{h^2}{2a^2} \left[ \frac{hS}{a} + \left( C_D - \frac{hS}{a} \right) \left( \frac{U_h}{U} \right)^2 \right] \quad (19)$$

where  $C_D = 2[U_{*h}/U_h]^2$  which is the gross drag coefficient of the vegetative canopy as seen by the surface boundary layer.

The definition of a drag coefficient for the surface boundary layer has proved fruitless in the past, since its magnitude depended very much upon the height at which the velocity was measured. However, it is seen that the drag coefficient defined for the top of the ideal canopy has special significance since  $(u_*/U)$  is constant within the ideal canopy.

In discussing equation (19), let us assume that  $S$  is constant with height within the canopy. From equation (12) we see that essentially we are assuming that the leaf-stalk area and drag coefficient functions are constant with height. This is in further agreement with the ideal canopy. On the basis of similarity, we shall assume that the mixing length is also constant with height within the canopy. With this condition, the following relationships result from equations (14) and (19):

$$\frac{hS}{C_D} = \frac{hAC_D^i}{2C_D} = \frac{(LSC)C_D^i}{2C_D} = a \quad (20a)$$

$$\frac{h^3 S}{2\ell_c^2} = \frac{h^3 AC_D^i}{4\ell_c^2} = \frac{h^3 (LSC)C_D^i}{4\ell_c^2} = a^3 \quad (20b)$$

$$\frac{C_D h^2}{2\ell_c^2} = a^2 \quad (20c)$$

$$S\ell_c = \frac{AC_D^i \ell_c}{2} = 2 \frac{C_D^3}{2} / a \quad (20d)$$

where  $LSC = hA$ . Our Leaf-Stalk Configuration term,  $LSC$ , is closely related to the Leaf-Area-Index ( $LAI$ ) often quoted by agronomists. The  $LAI$  is the total leaf area of a crop per unit of ground area. For a mature rigid agricultural crop, the  $LAI$  is usually very near to 3.5.

Equations (20a-20d) will be considered the basic relationships which characterize the "ideal canopy". These relationships could also have been derived directly from equation (12) if  $C'_e/L$  were made equivalent to  $\frac{1}{2}S$ . To reiterate, the ideal canopy is characterized by: (1) there is uniform vertical distribution of both the area density and the drag coefficient of the leaf-stalk configuration; (2) the drag coefficient of the leaf-stalk configuration is independent of local Reynolds number; (3) the mean velocity distribution within the canopy is exponential and there is (4) a constant mixing length and (5) a constant turbulence intensity existing within the canopy.

These characteristics of the ideal canopy should be compared with the "ideal" surface boundary layer as derived from the mixing-length hypothesis, that is, the logarithmic wind profile expressed by equation (4). In this case, the velocity profile is the opposite of that of the ideal canopy; that is, instead of being an exponential function, the velocity is a logarithmic function of height. The turbulence intensity ( $u_*/U$ ), instead of being constant, decreases rapidly with height; and the mixing length increases as a linear function of height.

In deriving the equations of the ideal canopy (20a-d), it was necessary to combine the characteristics of these two idealized models. It was assumed that the velocity and its first derivative, and the mixing length are continuous functions across the plane defined by the top of the canopy. On the basis of this assumption, it was possible to define the gross canopy drag coefficient  $C_D$  which is used in equations (20a), (20c), and (20d) to relate the canopy layer to the boundary layer. One of the consequences of this assumption is that the second derivative of the velocity and the first derivative of the mixing length are discontinuous functions through the plane defined by the canopy top.

The velocity profiles which result from the ideal canopy are as follows:

Above the canopy

$$\frac{U}{U_h} = 1 + \left(\frac{S\ell_c}{2}\right)^{1/3} \ln\left(1 + \frac{k(z-h)}{\ell_c}\right) = 1 + \left(\frac{C_D}{2}\right)^{1/2} \ln\left(1 + \frac{k(z-h)}{\ell_c}\right) \quad (21a)$$

Within the canopy

$$\frac{U}{U_h} = \exp\left[\frac{S}{C_D}(z-h)\right] = \exp\left[\left(\frac{S}{2\ell_c^3}\right)^{1/3}(z-h)\right] \quad (21b)$$

These equations are analogous to the logarithmic wind profile based on the roughness and zero-plane displacement in that three parameters are required to specify the wind profile equation (6). With regard to the above equations, these parameters are  $U_h$  with a combination of any two of the following:  $l_c$ ,  $S$  or  $C_D$ .

In section 2, it was noted that the roughness length and zero-plane displacement were not conservative properties of tall vegetation. The profile equations of the ideal canopy can be an improvement only if the parameters  $S$ ,  $C_D$ , and  $l_c$  are either conservative properties or in some manner uniquely interdependent for a given height, density, and type of vegetation. Equations (20a-d) are inadequate to substantiate this condition and thus recourse to experimental data is required.

### REAL CANOPIES

In the previous section, a hypothetical model for turbulent transfer within a vegetative canopy was derived. The comparison of this model to real canopy flows is severely limited by a paucity of data. However, some data are published and this will allow comparison of the ideal canopy with real canopies.

One of the basic assumptions of the ideal canopy was that the average leaf-stalk configuration is essentially invariant with height within the canopy. Figure 1, published by Allen, Yocum and Lemon (Reference 9), shows the accumulative Leaf Area Index (LAI) in a mature corn field indicating considerable uniformity of the height distribution of leaf area. Then as previously stated, the parameter  $A$  introduced in equation (14) is closely related to the volumetric leaf density which is the slope of the line shown in Figure 1. As can be seen, the slope is reasonably constant over a major portion of the canopy depth. Figure 1 represents only the leaf area; if the stalks and adventitious roots at the base and the tassels at the top were included, the plot of figure 1 would indicate less curvature and more closely approximate a straight line. In any event, it is not unreasonable, for a first approximation, to assume  $A$  to be constant throughout the canopy of at least a mature corn field.

Figure 2 depicts wind profiles above and within a corn field canopy as published by Tan and Ling (Reference 8). Only two of the profiles have data in the lower portion of the canopy. It is to be noted that the velocity profile is convex upward within the canopy except in the lowest 30 cm. The concave upward profile in the lower portion reflects the dominance of the ground surface over the vegetation in controlling the turbulence. Figure 3 is a plot of log of wind speed  $U$ , versus relative height within the canopy for the two complete profiles of figure 2. It is encouraging to note that the

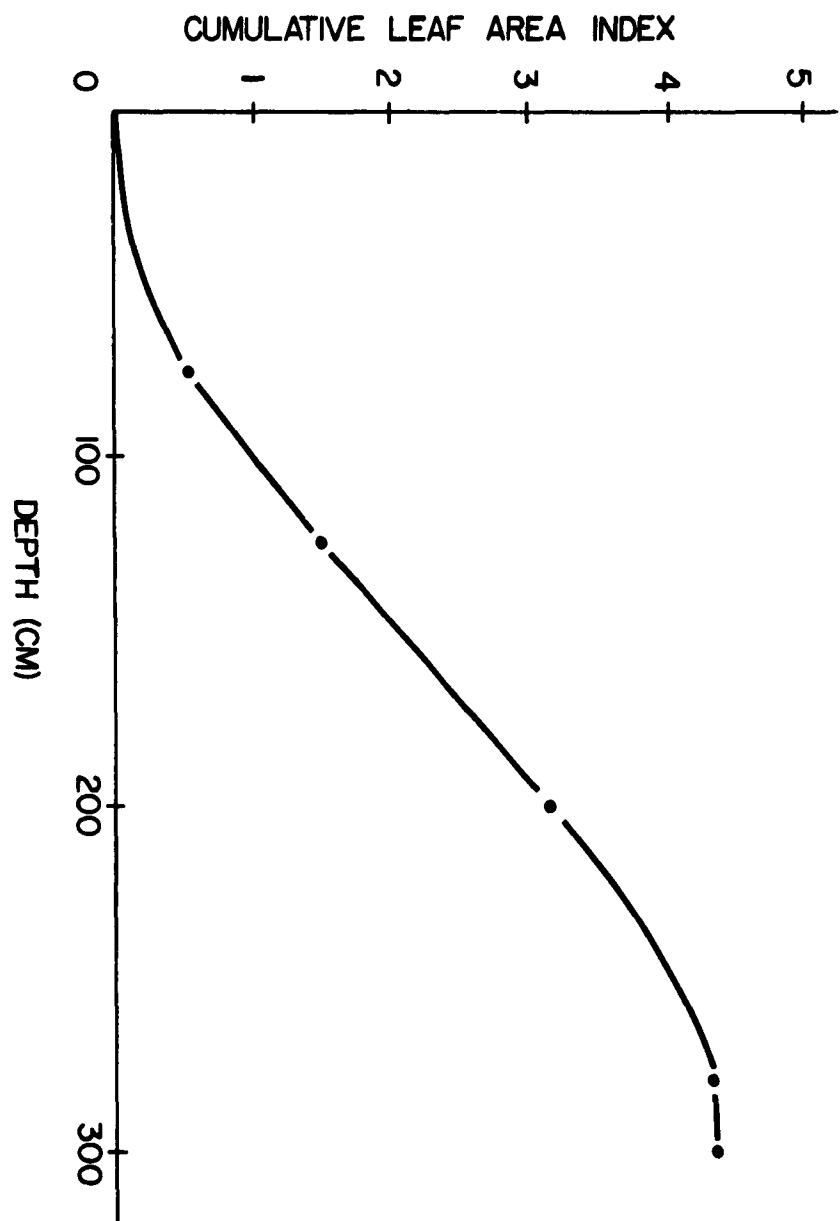
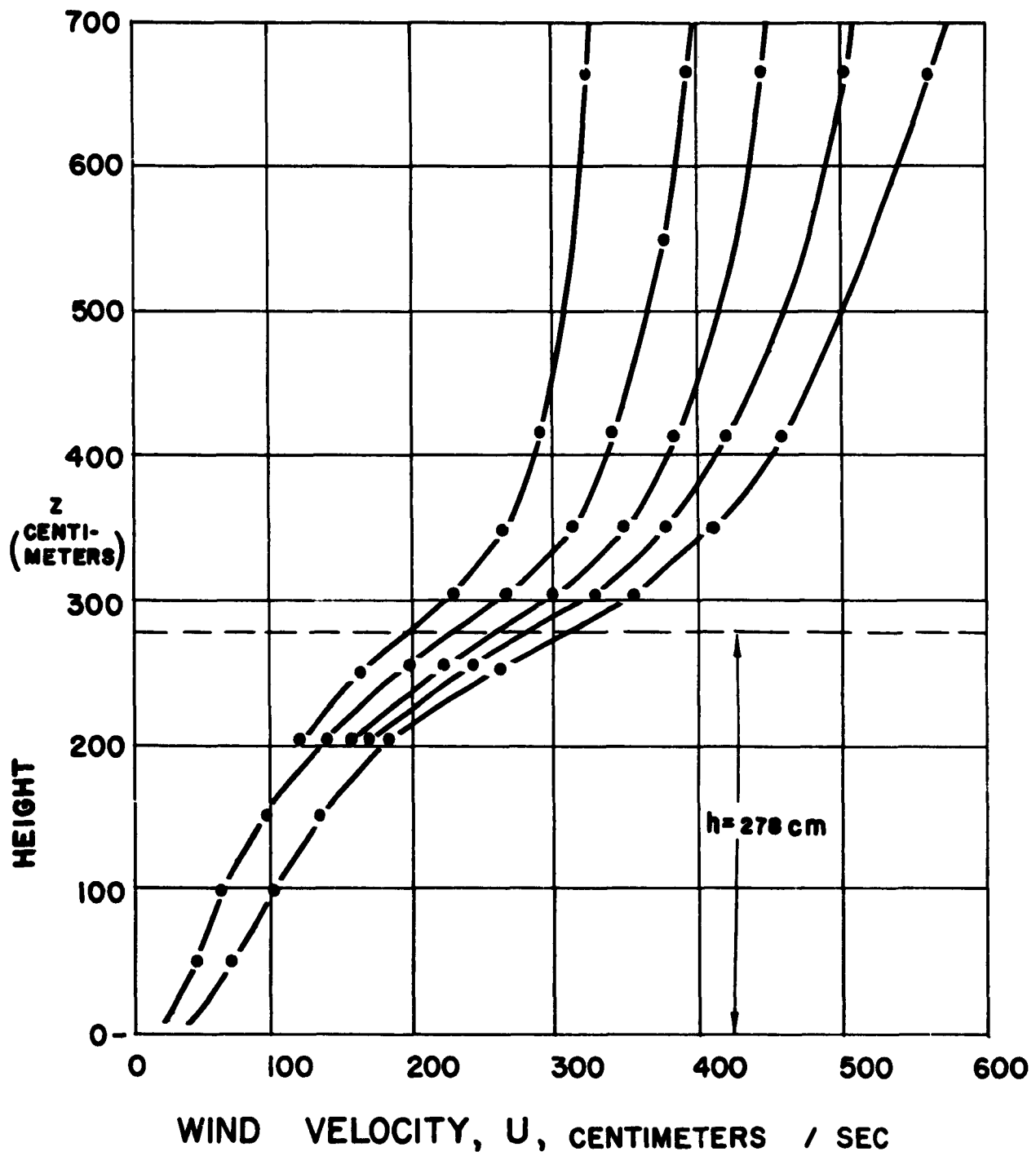


FIGURE 1. ACCUMULATIVE LEAF AREA AS A FUNCTION OF HEIGHT IN A MATURE CORN CANOPY. (REF 9)



**FIGURE 2.** WIND PROFILES ABOVE AND WITHIN A MATURE CORN CANOPY  
(REF 8)

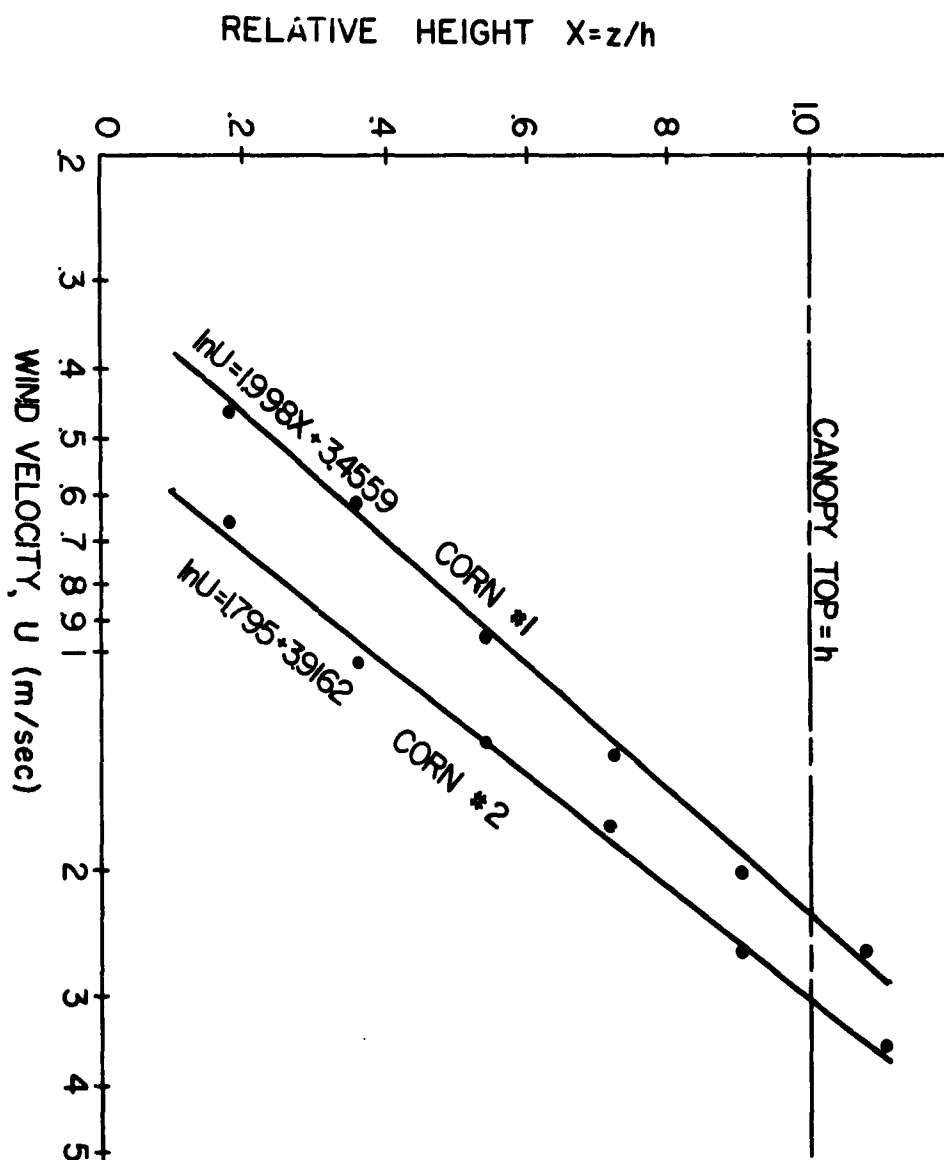


FIGURE 3. THE LOGARITHM OF THE WIND SPEED AS A FUNCTION OF THE RELATIVE HEIGHT WITHIN A MATURE CORN CANOPY.

data fit a straight line reasonably well. Thus, except for the region very near the ground surface, the velocity profile within corn, as reported by Tan and Ling, is exponential. The exponential profile leads to a constant turbulence intensity within the crop. Observations taken by Nakagawa (Reference 10) in a rice paddy canopy (see Table I) also show the turbulence intensity  $(u'^2)^{1/2}/U$  nearly constant.

TABLE I

Vertical Distribution of Horizontal Air Flow in A Rice Paddy - height of crop = 90cm. )after Nakagawa, Reference 10)

Z cm	$\bar{U}$ cm/sec	$\frac{(u'^2)^{1/2}}{U}$
40	12.3	0.321
55	13.6	0.346
70	22.3	0.331
85	25.7	0.320
100	52.1	0.331

According to equation (19) the condition of constant  $S$  within the canopy and an exponential velocity profile were necessary but not sufficient conditions to support the idea that the mixing length is constant within the ideal canopy. The verification of constant mixing length within the canopy from field data is somewhat arduous, but can be accomplished by the following procedure.

We are concerned with two distinct regions of turbulent flow - that within the canopy and that immediately above. We express the equations of momentum transfer in these two regions as follows:

$$\begin{aligned} \frac{\partial}{\partial z} \left[ K \frac{\partial u}{\partial z} \right] &= 0 && \text{Above the canopy} \\ \frac{\partial}{\partial z} \left[ K \frac{\partial u}{\partial z} \right] &= Su^2 && \text{Within the canopy} \end{aligned}$$

These equations can be combined into a single equation if we merely set  $S = 0$  for  $z > h$ , where  $h$  is the height of the canopy. With this in mind, we rewrite the above equations as:

$$\frac{\partial}{\partial z} \left[ l \frac{\partial u}{\partial z} \right]^2 = Su^2 \quad (21)$$

For computational purposes it is necessary to express equation (21) in non-dimensional terms. To do this we choose the mixing length and



velocity at height  $2h$  as reference values. With these, we define the following nondimensional variables:  $x = z/2h$ ,  $y = U/U_{2h}$ ,  $L = \ell^2/\ell_{2h}^2$ . Substituting these terms in equation (21) we obtain:

$$\frac{\partial}{\partial x} \left[ L \left( \frac{\partial y}{\partial x} \right)^2 \right] = \frac{8h^3 S}{\ell_{2h}^2} y^2 \quad (22)$$

We let  $B = 8h^3 S/\ell_{2h}^2$  and further differentiate so that equation (22) becomes:

$$\frac{\partial L}{\partial x} + 2L \left[ \frac{\partial^2 y}{\partial x^2} / \frac{\partial y}{\partial x} \right] = B \left[ \frac{y}{\frac{\partial y}{\partial x}} \right]^2 \quad (23)$$

Equation (23) retains the unknown parameter  $B$ . To eliminate this term we define a new variable  $E = L/B$ . Using this variable equation (23) can be expressed as follows:

$$\frac{\partial E}{\partial x} = \left[ \frac{y}{\frac{\partial y}{\partial x}} \right]^2 - 2E \left[ \frac{\partial^2 y}{\partial x^2} / \frac{\partial y}{\partial x} \right] = C_1 - 2E(C_2) \quad (24)$$

Equation (24) is the requisite equation to solve for the mixing length. It must be remembered that the variable  $C_1 = 0$  for  $z > h$ .

Equation (24) was solved for the two complete profiles of Figure 2. To do this it was necessary to determine the coefficients  $C_1$  and  $C_2$ . The number of actual data points contained in the profiles of Figure 2 is too few to adequately evaluate the velocity derivatives. Having no other guidelines, we accepted the profile curves of Tan and Ling as the best fit to the data. The portion of Figure 1 from the ground surface to  $2h$  (556cm) was subdivided by 20 equally spaced grid points and the velocities at each grid point were extracted from Figure 2. From those data, the velocity derivatives were estimated from Lagrangian polynomials. Once the coefficients  $C_1$  and  $C_2$  were determined, equation (24) was integrated from the ground surface to the height  $2h$  using a fourth-order Runge-Kutta integration routine.

Figure 4 is a graph of the nondimensional mixing length ( $\sqrt{E}$ ) for the two solutions of equation (24) reported by Cionco (Reference 11). In a general way, figure 4 is compatible with the ideal canopy. Within the canopy, the mixing length is essentially constant except near the ground surface; above the canopy the mixing length increases linearly with height. These conclusions are not as well supported by the solution for corn #2 as with corn #1. However, considering the experimental errors which may be involved, there is no reason to attribute significance to the deviations from the ideal in the corn #2 solution.

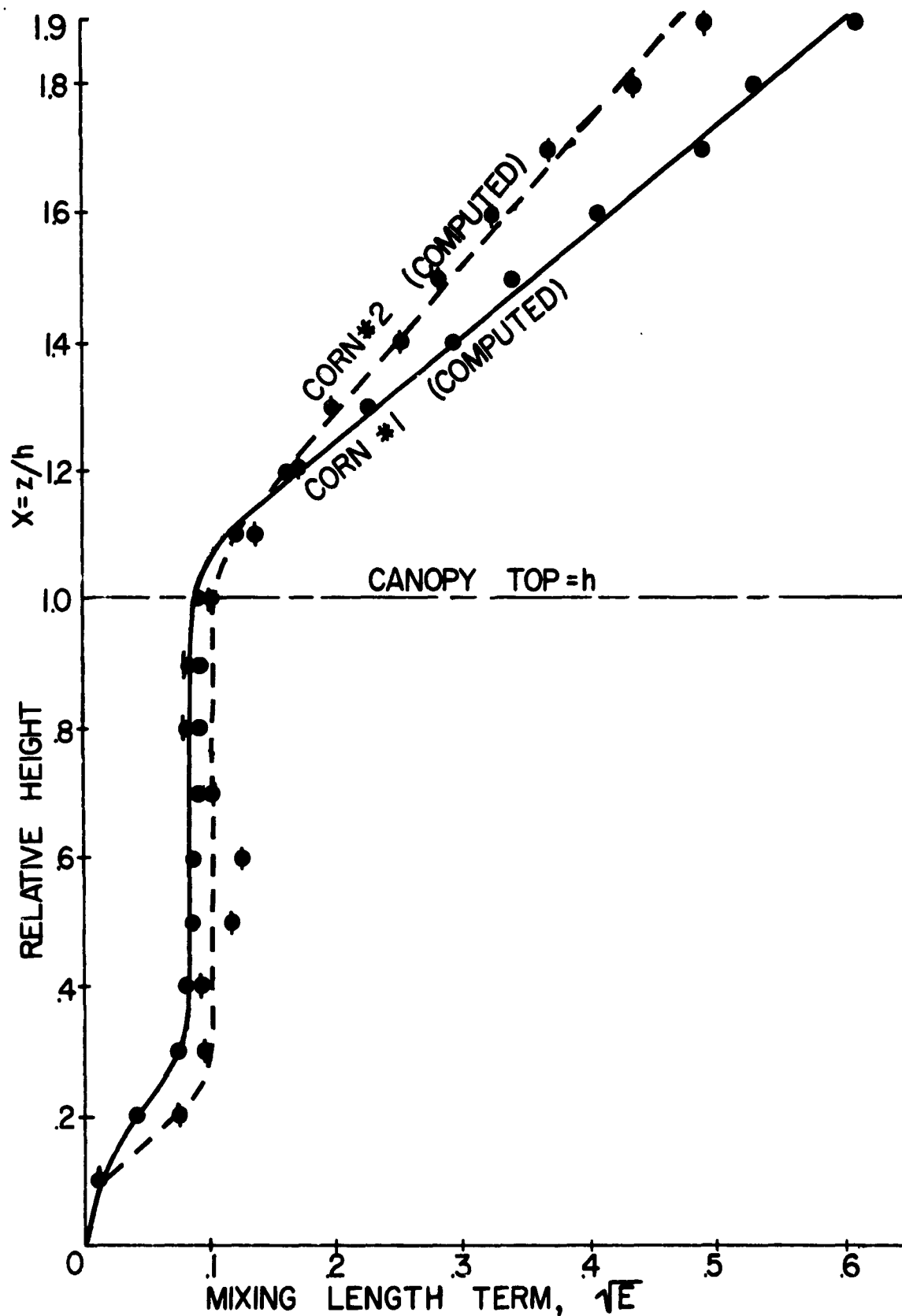


FIGURE 4. THE COMPUTED NONDIMENSIONAL MIXING LENGTH TERM AS A FUNCTION OF RELATIVE HEIGHT WITHIN AND ABOVE A MATURE CORN CANOPY. (REF 11)

Figures 1, 3, and 4, in a qualitative manner, agree with the general characteristics of the ideal canopy. To quantitatively evaluate the equations (20a-d), it is essential to compute the values of  $S$ ,  $C_D$  and  $\ell_c$ . This can be done by assuming that the proportionality factor between the mixing length and the height is von Karman's constant. The nondimensional mixing lengths were equated to linear functions of height with least-squared error incorporating all values of  $\sqrt{E}$  above the canopy. These are the straight lines shown in figure 4. In addition, the average value of  $\sqrt{E}$  within the canopy (excluding the region near the ground) was computed. With these relationships the values of  $S$ , the zero-plane displacement  $D$ , and  $\ell_c$  can be determined. The velocity ratio  $U/U_h$  is proportional to  $\ln(Z - D)$  by the factor  $(C_D/2)^{1/2}$ ; thus,  $C_D$  can be computed from a least squared error linear function relating  $U/U_h$  to  $\ln(Z - D)$ . Table II contains the computed values for the two corn profiles of the requisite parameters of the ideal canopy.

TABLE II

	$h$	$D$	$a$	$C_D$	$S$	$\ell_c$	$k(h-D)$
Corn #1	278cm	257.2cm	2.000	$2.462 \times 10^{-2}$	$1.875 \times 10^{-4}$	15.83cm	8.32
Corn #2	278cm	227.8	1.795	$4.370 \times 10^{-2}$	$3.739 \times 10^{-4}$	27.26cm	24.08

It is immediately apparent that  $C_D$ ,  $S$ , and  $\ell_c$  are not conservative properties of a mature corn canopy. Like the roughness length, these parameters increase with increasing wind speed; however, it is easily shown that individually their relative increase is less than that of the roughness. On the other hand, the most conservative property of the canopy appears to be  $a$ , the derivative of  $\ln U$  with respect to relative height  $z/h$  within the canopy.

TABLE III

	$a$	$hS/C_D$	$(h^3 S / 2 \ell_c)^{1/3}$	$S \ell_c$	$2(C_D/2)^{3/2}$
Corn #1	2.000	2.115	2.020	$2.97 \times 10^{-3}$	$2.73 \times 10^{-3}$
Corn #2	1.795	2.375	1.753	$1.019 \times 10^{-2}$	$.646 \times 10^{-2}$

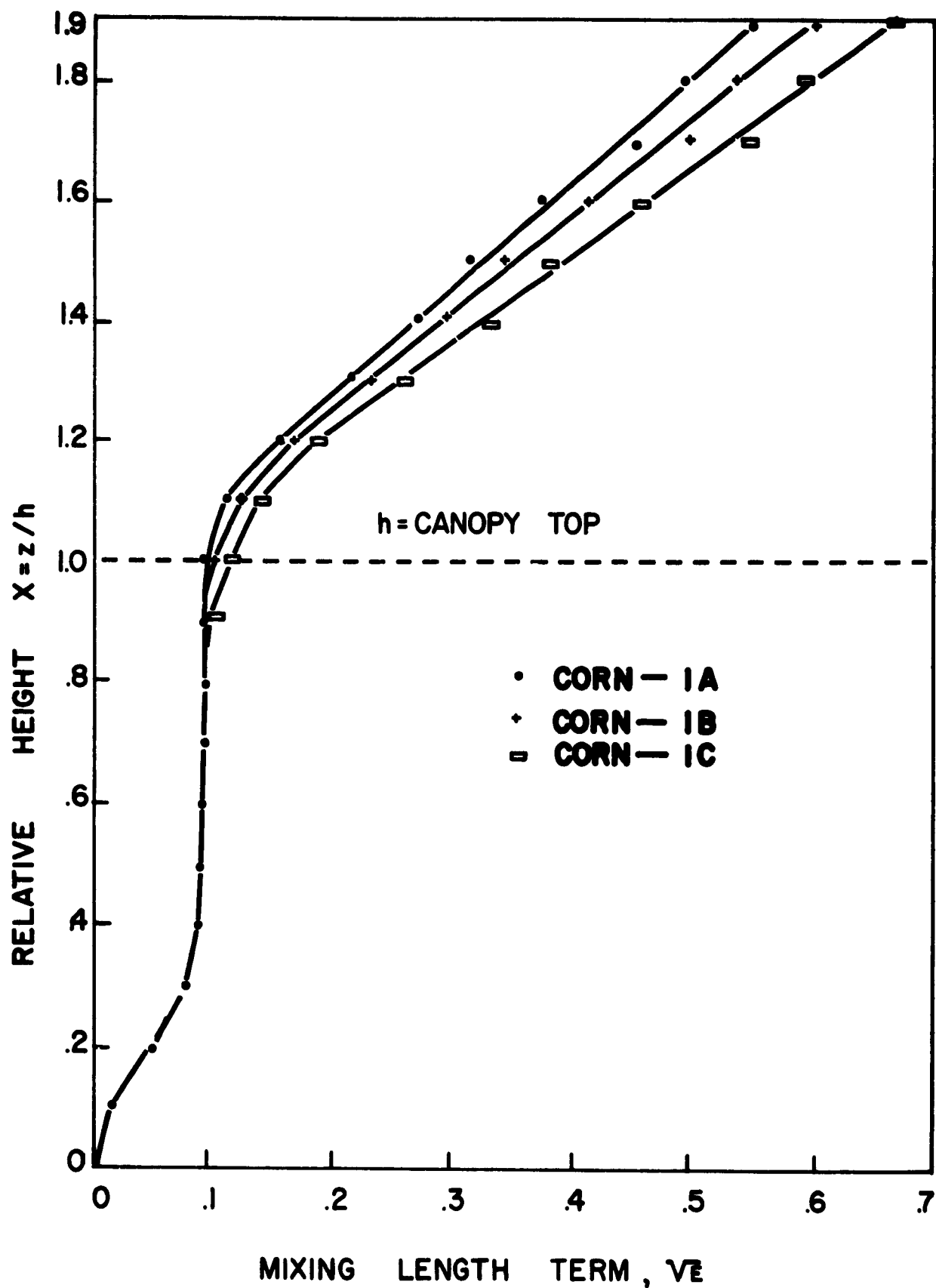
In Table III, the numerical values for terms involved in equations (20a,b,d) are presented. Although the values are close to those defined in the ideal canopy, it is evident that the values of  $C_D$  are consistently too small. This failure of the ideal canopy is due to the manner in which the ideal canopy was joined with the logarithmic wind profile of the surface boundary layer at the top of the canopy.

In the above computations it was assumed that the canopy top was 278cm—the figure given by Tan and Ling. From a practical standpoint, it must be somewhat difficult to define the "top" of a corn field. If a lower reference plane had been chosen,  $C_D$  would have been larger, and consequently the figures of Table II would likely be more compatible. Nevertheless, it is apparent that one of the difficult problems associated with the canopy approach is the manner in which the ideal canopy is joined to the logarithmic wind profile. In developing the ideal canopy, we did not consider any variation of  $S$  with height, and it is at the top where this is likely to be most significant. This is well illustrated in Figure 5 which presents solutions of corn #1 for various assumed shapes of the  $S$  function at the canopy top. Table IV lists the magnitudes of the canopy parameters which result from these profiles. These results clearly reveal that the mixing-length profile is sensitive to changes in shape of the  $S$  function and that  $l_c \neq k(h + D)$ .

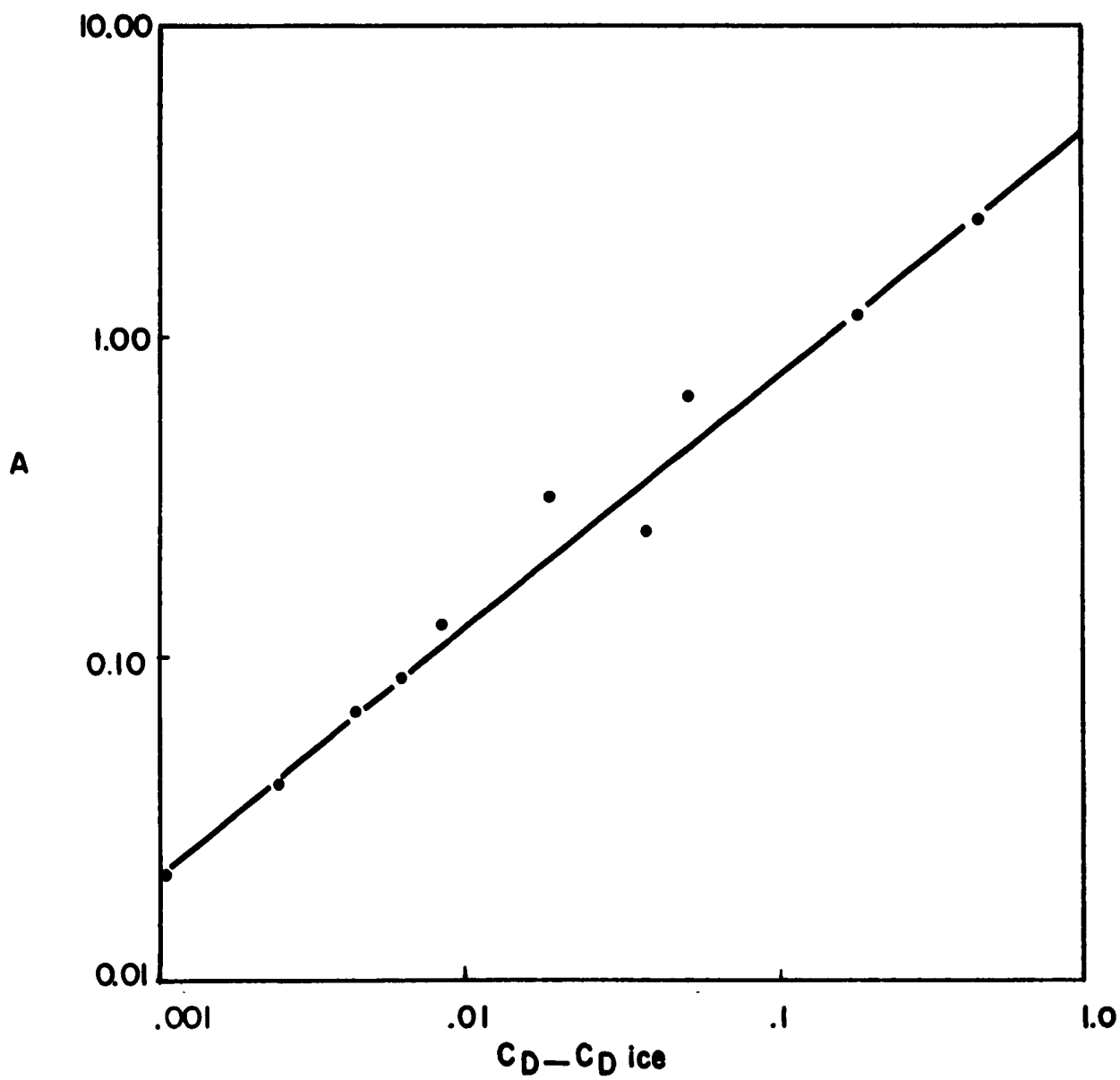
	<u>TABLE IV</u>				
	$S$	$l_c$	$C_D$	$hs/C_D$	$a$
Corn #1A	$2.230 \times 10^{-4}$	17.26	$2.462 \times 10^{-3}$	2.52	2.00
Corn #1B	$1.875 \times 10^{-4}$	15.83	$2.462 \times 10^{-3}$	2.115	2.00
Corn #1C	$1.550 \times 10^{-4}$	14.39	$2.462 \times 10^{-3}$	1.75	2.00

The results for corn have clearly shown that  $S$  and  $l_c$  are not conservative properties since both increase with increasing wind speed. The increase of  $S$  with wind speed might be attributed to increased effectiveness of the drag of the leaves due to flapping. However, this appears to invalidate the assumption that the local drag coefficient of the vegetation is independent of Reynolds number. This does not invalidate the ideal canopy but it does complicate the equations. Annex A contains the derivations of equations analogous to equations (20a) thru (20d) which include a local Reynolds number effect. They show that the mixing length and turbulence intensity cannot be constant within the canopy. Since Tan and Ling's data taken in mature corn and Nakagawa's data taken in a rice paddy both show a constant turbulence intensity within the canopy, it was assumed the Reynolds number effects could be neglected for crops of this configuration, structure, and elasticity. Undoubtedly, these features are related to the maturity of the two crops and the assumption may prove invalid for less mature fields.

It is interesting to investigate the density variation of roughness elements on canopy characteristics. The only quantitative data available for this purpose is the bushel basket data of Kutzbach (Reference 3). Figure 6 shows the relationship between the density



**FIGURE 5.** SOLUTIONS OF THE MIXING LENGTHS IN MATURE CORN AS A FUNCTION OF THE SHAPE FACTOR,  $S$ , FOR CORN #1.



**FIGURE 6.** RELATIONSHIP BETWEEN THE BUSHEL BASKET DENSITY AND THE DRAG COEFFICIENT

function  $A^{++}$  with the drag coefficient at basket height less the drag coefficient at basket height associated with only the ice surface. This correction for the effect of the ice is important only at small values of  $A$ . The least squared error analysis results in the following relationship:

$$C_D - C_{D_{ice}} = 3.904 \times 10^{-4} A^{1.243} \quad (25)$$

Utilizing equation (14), we can write equation (20d) in the form  $AC_D' \ell_c = 2 (C_D/2)^{3/2}$ . At first thought it might seem reasonable to assume that the local drag coefficient ( $C_D'$ ) of a rigid bushel basket would be a constant, but combining this equation with equation (25) leads to the result that  $C_D' \ell_c \propto C_D^{0.7}$ . If  $C_D'$  were a constant,

this would lead to the ridiculous result that  $\ell_h$  would vary more than an order of magnitude over the range of  $A$  given in figure 6. We can conclude that  $C_D'$  is not independent of the density function  $A$  that is the drag of the baskets increases with increased packing. This is qualitatively in agreement with the results indicating that compact 75cm high alfalfa is a rougher canopy than loose 278cm high corn. As a consequence, only the parameter  $S$  has significance with regard to the ideal canopy.

#### DISCUSSION

In brief, the problem of the canopy flow study was to develop a canopy flow model from basic parameters. The ideal concept told us that the model is characterized by the following properties within the canopy for the steady condition and no advection:

- (1) the mixing length is constant,
- (2) the mean velocity distribution is exponential and,
- (3) the turbulence intensity is constant.

Further development yielded a set of four equations that are considered the basic relationships that characterize the ideal canopy. In combining these characteristics with those of the "Ideal Surface Boundary Layer" of the mixing-length hypothesis, the development yielded two wind profile equations that relate the canopy to the

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<sup>++</sup>The  $A$  is not Kutzbach's  $A$ , but rather is the reciprocal of the specific area defined by Kutzbach.

boundary layer by means of the total drag coefficient of the canopy. The equations dictate that the canopy wind profile is an exponential function of height.

It was entirely fortuitous that the first attempt to verify the ideal canopy met with at least qualitative success utilizing the data for corn published by Tan and Ling. Obtaining solutions of nearly constant mixing length profiles throughout most of the crop at the onset of the study provided much encouragement. However, there are other published data which are either partially or totally incompatible with the ideal canopy as to the distribution of velocity within the canopy. Undoubtedly there are many reasons for this, a few of the more obvious being:

- (1) failure to meet the assumptions of uniform distribution throughout the canopy,
- (2) neglect of the Reynolds number effects,
- (3) omission of terms in the transport equations,
- (4) inadequate sampling in the observations and,
- (5) topographic and natural surfaces irregularities at or surrounding observation site.

Figures 7a-c present data from the literature for different vegetative covers and the double bushel basket experiment of Kutzbach. The various vegetative covers represented are rice paddy, brush, timber forest, sugar beets, wheat, corn, and citrus orchard. In general, it is evident that "a" of equation (15) is not constant in the canopy as proposed by the ideal canopy. However, in some vegetation the variation is small. A class of canopy profiles which is entirely incompatible with the concept that the momentum transport is proportional to the velocity shear is represented by those of Fons (Reference 12) in brush, and those of Lemon and Stoller (Reference 5) in immature corn and wheat. These profiles are characterized by an extended region of the canopy profile where shear is negligible. This class of canopy profiles implies an additional term in the turbulent transfer equation.

The solution for the mixing length in the high wind speed case for the corn data of Tan and Ling (Reference 8) also might be construed to indicate an additional term. In the solution, the transport of momentum is accounted for completely by  $\partial u / \partial z$ . But for this group of profiles the velocity shear is nearly zero throughout most of the crop, thus the mixing length must vary within the canopy to balance the equation. The mixing length for the corn #2 solution indicates a slight maximum in the profile within the crop. This tendency has also been observed in mixing-length analyses made by



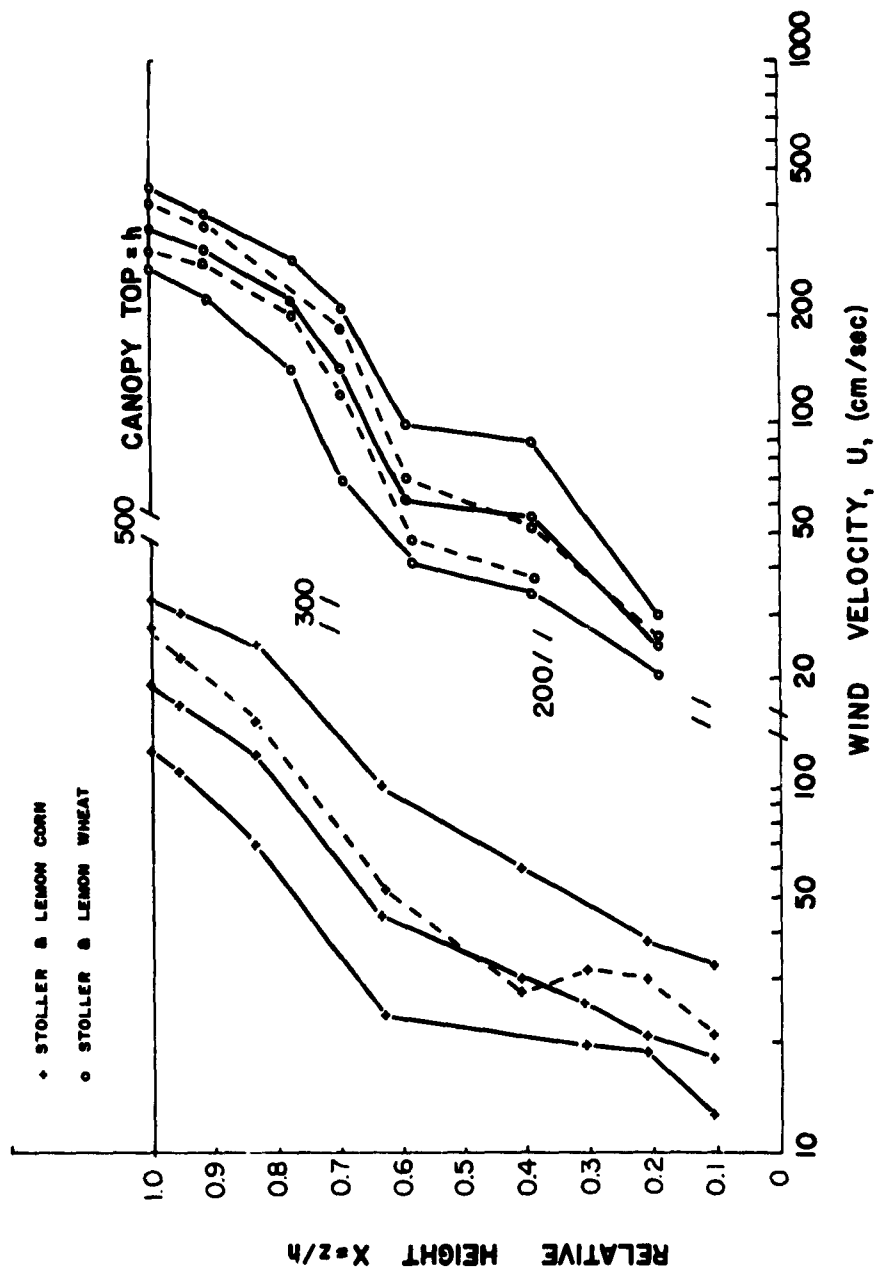


FIGURE 7A. OBSERVED WIND PROFILES IN VARIOUS VEGETATIVE CANOPIES.

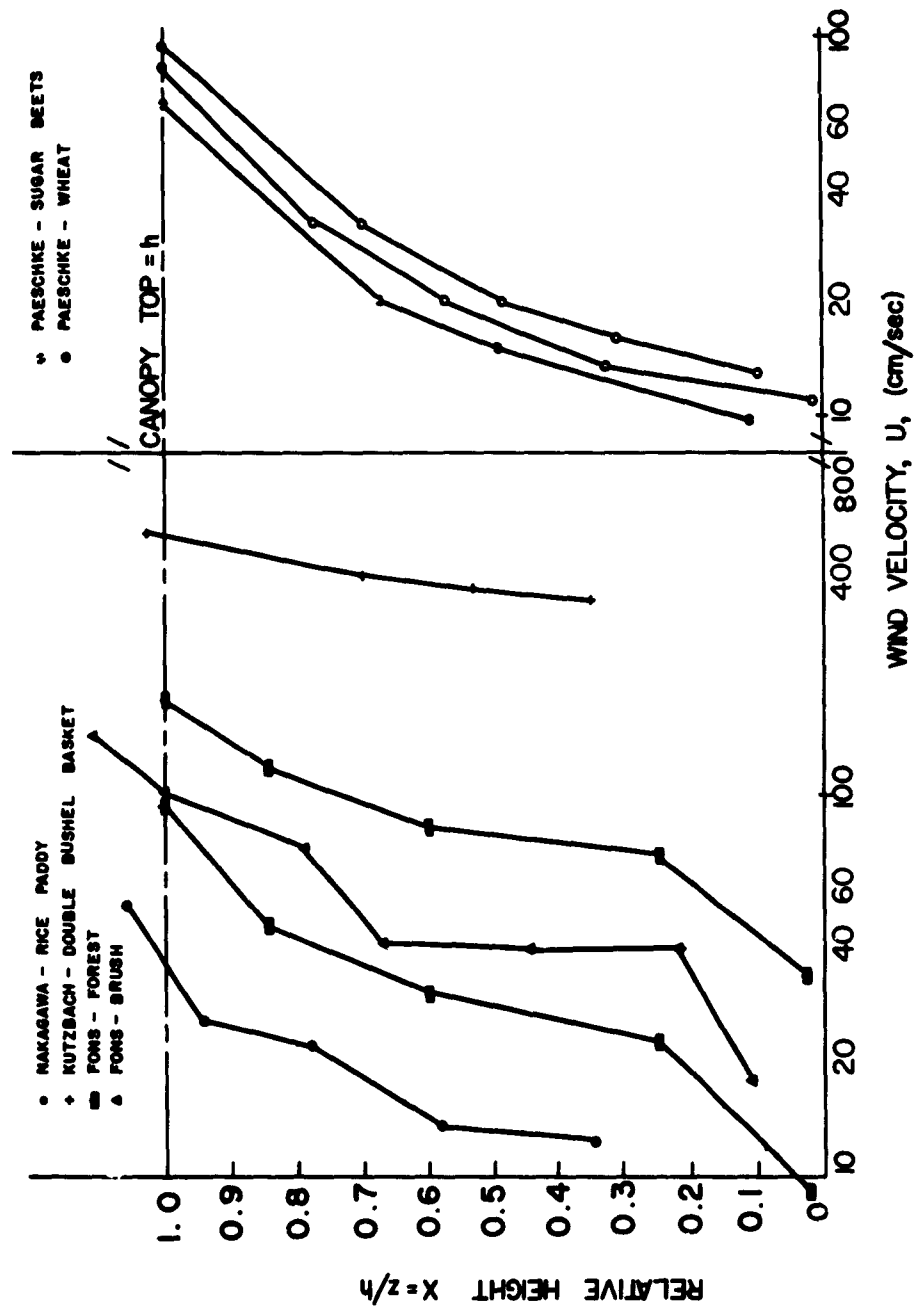


FIGURE 7B. OBSERVED WIND PROFILES IN VARIOUS VEGETATIVE CANOPIES.

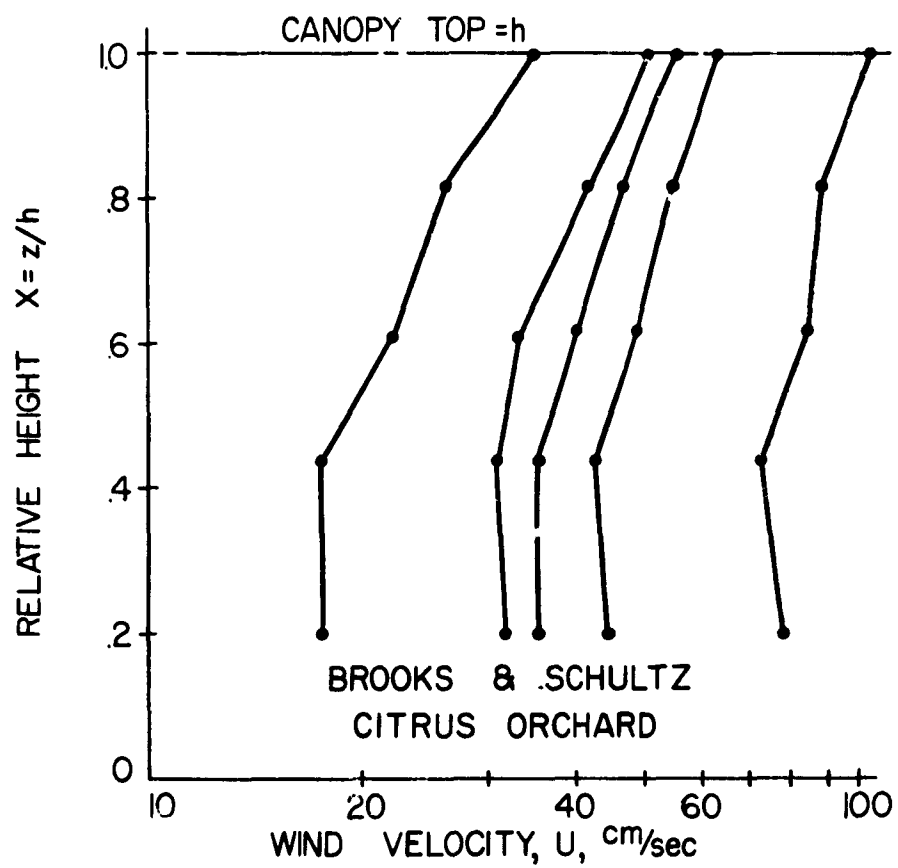


FIGURE 7C. OBSERVED WIND PROFILES IN VARIOUS VEGETATIVE CANOPIES.

Uchijima (Reference 14). This type profile can be qualitatively reconciled with the general theory of turbulent transport, but the requisite data are nonexistent.

Intuitively one would expect a considerable amount of turbulence to be generated in the upper portion of the canopy as the wind encounters the crop. This would mean that  $q^2 = (U'^2 + V'^2 + W'^2)w$  would be fairly large in the upper portion of the canopy. Since this term must be zero at the surface, the gradient within the mid portion must be significant. How wire anemometer measurements made by Stoller (Reference 5) seem to indicate this is true. If one makes an analogy between the second order momentum fluctuations with the second order temperature fluctuations as discussed by Deardorff (Reference 15), it would appear that the flux due to the diffusive term  $\frac{\lambda}{\lambda z} \left( \frac{1}{2} q^2 w + \frac{1}{\rho} p w \right)$  is important especially at moderate wind speeds.

For those crops that are especially uniform in configuration, the mixing length should be quite restricted in its growth and be fairly constant throughout most of the crop. The question may arise as to why a unique mixing length does not exist within the canopy, especially when it is emphasized that the ideal canopy has a uniform distribution of the area density and the drag coefficient of the leaf-stalk configuration. The plant configuration should also restrict the size of the eddies. If the vertical area distribution and the physical spacing of the plants are the only controlling factors of the size of the eddies, then the mixing length within the canopy should remain fixed regardless of the wind speed. However, the solutions for the mixing length in the real canopy show nearly a two fold increase for an increase in the wind speed of 233cm/sec to 317cm/sec at the canopy top. One should expect an increasing mixing length since the resulting basic relationships of the ideal model indicate that the mixing length in the crop is a function of the coefficient of the total drag force being exerted upon the canopy volume. The drag in turn is a function of the air flow distribution above the canopy, which governs the degree of penetration of air flow down into the canopy. The analysis of the velocity profiles above the cornfield using equation 6 shows that the zero-plane displacement decreased with increasing wind speed. This is also true of a wheat field - up to a critical wind speed. Apparently the opening of the canopy occurs as the increasing wind speed orients the leaves along the flow. The observations indicate that the mixing length apparently increases uniformly throughout the crop. However, at some greater speed the drag exerted on the canopy reaches a critical value and remains somewhat constant for greater velocities. In turn the size of the eddies must reach a maximum value similar to that imposed by the plant spacing and vertical area distribution. Therefore, rather than

expect a unique mixing length for the various real canopies, one should be prepared to accept a mixing length in the crop that is a function of the air flow above the canopy but with a limiting critical value.

This study is an exploratory attempt to understand the turbulent transfer within a vegetative canopy. The intent in developing the ideal canopy was heuristic. It is hoped that this report will stimulate thought and investigation which will ultimately lead to an expression of the aerodynamic roughness effects in terms of vegetative characteristics.

### CONCLUSIONS

Turbulent transfer within real vegetative canopies is in general not ideal. The utility of the canopy approach has not been proven, but it does seem to show promise. The relationships developed can be applied to vegetative covers with characteristics that approach the assumptions of the ideal canopy with reasonable success.

The character of the mixing length and the parameter  $S$  with regard to the density and structure of the vegetation and its relationships to the Reynolds number should be investigated. It may be possible to do this in a wind tunnel or with controlled experiments such as the bushel basket or in real vegetative canopies. The manner in which the canopy flow joins the surface boundary layer flow at the top of the canopy is another major problem requiring investigation. But, foremost, the theoretical approach must be experimentally verified.

Future development of the ideal concept requires the collection of a set of data on the mean velocities and turbulent structure within and above the canopy.

## ANNEX A

### REYNOLDS NUMBER EFFECTS

Previous investigations and our work up to this point have assumed that the flow within the canopy is independent of local Reynolds number. In certain types of vegetation or within certain limits of wind speed, the assumption may be valid. However, if one considers extremely limber vegetative covers such as alfalfa this assumption may be erroneous. The fact that  $S$  and  $l_c$  are not conservative properties appears to invalidate the assumption that the local drag coefficient is independent of Reynolds number. As was pointed out, this does not invalidate the ideal canopy, but it does complicate the equations. To show this, let us define a local Reynolds number in the canopy as  $Re = U/\nu$  and express the parameter  $S$  as  $S = sRe^n$ , where  $s$  is a constant and  $\nu$  is the kinematic viscosity. Thus, equation (19) becomes:

$$l^2 = \frac{h}{a^2} \left[ \frac{hsRe^n}{(2+n)a} + \left( \frac{C_D - hsRe^n}{2} \right) \left( \frac{U_h}{U} \right)^2 \right] = \frac{h^3}{a^2} \left[ \frac{hS}{(2+n)a} + \left( \frac{C_D}{2} - \frac{hS_h}{(2+n)a} \right) \left( \frac{U_h}{U} \right)^2 \right] \quad (26)$$

Equation (26) is analogous to equation (19), but it has the notable difference that it is not possible for the mixing length and the turbulence intensity ( $u_*/U$ ) to be constant with height. If once again we assume a dynamic similarity between  $l^2$  and the function  $S$ , then the following conditions result from equation (26):

$$\frac{hS_h}{C_D} = \frac{hs}{C_D} \left( \frac{Reg}{LSC} \right)^n = \frac{(2+n)a}{2} \quad (27a)$$

$$\frac{h^3 S}{(2+n)l^2} = a^2 \quad (27b)$$

$$\frac{h^3 C_D}{2l_h^2} = a^2 \quad (27c)$$

$$s_h l_h = l_h s \left( \frac{Reg}{LSC} \right)^n = (2+n) (C_D/2)^{3/2} \quad (27d)$$

where  $Reg$  is the gross canopy Reynolds number defined as  $Reg = (hU_h/\nu)$ . Equations (27a-d) are very similar to equations (20a-d); the most significant difference is the inclusion of the factor  $(2+n)$  in place of 2. Because of their greater generality, we shall now assume that the ideal canopy is defined by equations (27a-d).

Our previous analysis was based upon only two profiles in a corn field. Most available data deal only with the flow above the canopy and the results are usually published in terms of the friction velocity, zero-plane displacement and roughness length. It is possible to put this form of data into use if we assume  $l_h = k(h-D)$  and  $C_D = 2(k/\ln(h-D)/z_0)^2$  and determine a functional relationship between  $\ln l_h$  and  $\ln C_D$  and between  $\ln C_D$  and  $\ln U_h$ .

The data for corn in Figure 2 as published by Tan and Ling (Reference 8) and data for alfalfa published by Stoller and Lemon (Reference 5) were analyzed in this manner. Least-squared error linear functions were evaluated with the following results:

$$l_h = 687.2 (C_D/2)^{1.1236}; C_D/2 = 1.066 \times 10^{-8} U_h^{2.01} \quad \text{CORN}$$

$$l_h = 48.48 (C_D/2)^{.6184}; C_D/2 = 1040.5 U_h^{-2.027} \quad \text{ALFALFA}$$

If these equations are incorporated into equation (27d), the following relationships result:

$$S_h = 3.8658 \times 10^{-3} (C_D/2)^{.3764}; S_h = 2.6638 \times 10^{-3} U_h^{.7566} \quad \text{CORN}$$

$$S_h = 5.014 \times 10^{-3} (C_D/2)^{.8816}; S_h = .9251 U_h^{-1.787} \quad \text{ALFALFA}$$

These results clearly reveal the striking difference that exists in drag characteristics between a timber, dense canopy such as alfalfa and a semi-rigid open canopy such as corn.

Admittedly the method of analysis is crude. However, the results should represent qualitative indications of the Reynolds number effects. The  $n$  value for corn is of special interest since one usually expects the Reynolds number exponent to be a negative value. It is obvious that more comprehensive measurements of the wind flow in and over vegetative canopies must be accumulated before the Reynolds number effects can be expressed in a quantitative manner.

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